Meeting Material Specs

Relating Process Capability Indices and the Precision-to-Tolerance Ratio

By Joel Dobson

Q Is there an important connection between the process capability indices (CPU and CPL) and the precision-to-tolerance ratio?

A The probability of a true value being out of specification, given that the measured value is in spec, depends on the values of the process capability indices, CPU and CPL.

Two processes with the same PTR can have different probabilities of shipping out of spec material because of different process capability values.

When judging gauge repeatability and reproducibility (GRR), some use a criterion based on the PTR. We define PTR using:

$$PTR = \frac{6 * \sigma_{RR}}{(USL - LSL)}$$

Section 10.5.12 of the standard guide for measurement systems analysis (E2782), mentions this method but gives no decision criterion based solely on PTR. Instead, the guide gives criteria for decisions based on the probability of acceptance given the measurement value in Section 10.7. In this article, we show two examples with the same PTR but differing probabilities of the true values being out of spec, given the measured values are in spec. We will demonstrate that process capability indices, CPU and CPL, are related to this probability.

EXAMPLE 1

Suppose we have the following conditions: {LSL = 5, USL = 15, μ of the measurements = μ_m = 12, CPU = 0.9, and PTR = 0.301, which is the same as %GRR = 30.1%}. We estimate that:

measurement $\sigma = \sigma_m$ = 1.11111,

CPL = 2.1,

σ_{RR} = 0.5016667, and

σ of the true values = σ_t =

$$\sqrt{1.1111^2 - 0.5016667^2} = 0.991412$$

We assume that the true values (X1) and the measured values (X2) form a bivariate normal distribution, with mean (12, 12) and covariance matrix with elements: variance(X1) = 0.9828985, variance(X2) = 1.23456789, and covariance(X1, X2) = 0.9828985. This comes from the idea that the measured value (m or X2) is the sum of the true, unknown value (t or X1) and an error term with mean zero (e), or X2 = X1 + e. We further assume that X1 and e are independent and normally distributed.

From this bivariate normal distribution, the probability estimate is 3.922e-04 that the true reading is out of specification, given that the measured reading is in spec. This is 392.2 parts per million. We could consider this the probability of shipping out of spec material caused by gauge imprecision, which is related to the customer's risk. We used the R function pmvnorm() from the R library mvtnorm to make this calculation.

EXAMPLE 2

Under different conditions of {LSL = 5, USL = 15, μ of the measurements = μ_m = 10, CPU = 1.67, and PTR = 0.301}, we estimate that:

measurement $\sigma = \sigma_m$ = 0.998004,

CPL = 1.67,

σ_{RR} = 0.5016667, and

σ of the true values = σ_t =

$$\sqrt{0.998004^2 - 0.5016667^2} = 0.8627529$$

We assume that the true values (X1) and the measured values (X2) form a bivariate normal distribution, with mean (10, 10) and covariance matrix having elements: variance(X1) = 0.7443425, variance(X2) = 0.9960120, and covariance(X1, X2) = 0.7443425. From this bivariate normal distribution, we estimate the probability of any true reading being out of spec, given that the measured reading is in specification, as 2.690638e-09. This is about 3 parts per billion.

"Two processes with the same PTR value can have different probabilities of shipping out of specification material, because of the different process capabilities values."

CONCLUSION

From these two examples, we conclude that two processes with the same PTR value can have different probabilities of shipping out of specification material, because of the different process capabilities values.

The standard guide for measurement systems analysis (E2782), mentions that some GRR judgments are based on PTR, but the criteria for such judgments are not specified in the guideline. The guideline suggests decisions using the probability of acceptance given measurement value, instead of using the PTR. Given that differing processes with the same PTR can give differing customer risks, this may be a better approach.

Send your statistics questions to Maryann Gorman at mgorman@astm.org.

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NEXT GEN

STANDARDS IN EDUCATION

Drexel University students visited ASTM in March to study the standards development process as part of a new class made possible through a standards education grant from the U.S. National Institute of Standards and Technology. In addition to the students and faculty from Drexel University as well as ASTM staff, NIST and four visiting international standards experts also attended.

Nominations Sought for Professor of the Year

ASTM International is currently accepting nominations for its 2017 Professor of the Year Award. The award recognizes and rewards the contributions of educators in developing students' understanding of standards. Both the winning professor and their university will receive a $2000 honorarium. Interested parties are requested to submit nominations by August 31, 2017. Three independent references are also required. Please visit www.astm.org/studentmember/Professor_Year.html for more details.

Understanding that engaging and providing tools for university professors has been the most effective way to reach the next generation of technical experts enrolled in universities worldwide, ASTM continues to develop new resources to integrate information on technical standards into existing engineering curricula. Educators can use the resources available on ASTM's Students and Professors pages (www.astm.org/campus) to expose students to the dynamic field of standardization.

Project Grants Available

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