Building on Reliability: Reliability Test Planning, Part 2

By Stephen N. Luko and Dean V. Neubauer

Q: How does a Weibull statistical model help in reliability test planning?

A: In our first article on this topic, we discussed how a test plan works: the test parameters, the type of data that results, and certain cases that are called nonparametric in that they do not depend on any assumed distribution for the failure mode. In this part, we build further on these concepts and add to the mix the assumption of a Weibull statistical model for the underlying failure mode.

The Weibull distribution is a widely used model for failure phenomena in electromechanical components, systems, and materials testing. It is capable of modeling the three major classifications of failure modes that are commonly found: a) random, b) wear-out or degradation, and c) early type failures called “infant mortality.” Moreover, these three classifications are related to specific values of the Weibull distribution shape parameter, β. When failures are random, β = 1; when failures are wear-out or degradation related, β > 1; and when failures are infant mortality-related, β < 1.

Often, the starting point for reliability requirements demonstration is to assume β = 1. In the case of wear-out/fatigue types of failure modes, 1 < β ≤ 4 is commonly found. During an early product release/introduction or following some product modification, infant mortality conditions might occur, resulting in early failure for some units. In these cases, β < 1. This would typically occur in field applications. Generally, we do not test/validate for the infant mortality case, although in theory it is possible, as smaller values of β generally mean more variation in the life of a product.

When designing a test plan for the Weibull case, we have to assume a value of β. Generally, there may be good reason for any choice as there may be data (history) to draw from within a specific company, or there may be some public domain acknowledgment of a range of possible β values within a specific industry for a specific type of failure mode.

The standard two-parameter Weibull model has a closed-form cumulative distribution function described as:

\[ F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta} \]  

(1)

In Equation 1, \( F(t) \) is the cumulative failure probability at time \( t \); \( \beta \) is the Weibull shape parameter (also called the Weibull slope) discussed earlier; and \( \eta \) is the Weibull characteristic life, or 63.2th percentile of the Weibull distribution model. Equation 1 is used, along with other theory, in developing test plans for the Weibull model.

RELIABILITY DEMONSTRATION PLANNING: WEIBULL MODEL, ZERO FAILURES

Recalling that if \( n \) units are tested for a time \( t \) and zero failures result, then using confidence \( C \), the sample size requirement to demonstrate a reliability, \( R \), would be:

\[ n = \frac{\ln(1 - C)}{\ln(R)} \]  

(2)

If the Weibull distribution is an assumption, Equation 2 is modified as shown in Equation 3:

\[ n = \frac{\ln(1 - C)}{m^\beta \ln(R)} \]  

(3)

In Equation 3, the variable \( m \) is the ratio of test time \( t \) to the time \( y \) at which reliability \( R \) applies. It is the “number of lives” the test time \( t \) is equivalent to, where a “life” is the life having reliability \( R \). The assumed Weibull shape (slope) parameter is also needed. Equation 3 comes from the fact that there are zero failures allowed, and it uses the fundamental nonparametric relationship among reliability, confidence, and sample size, such that \( R^n \geq 1 - C \). Then substituting the Weibull reliability function for \( R(y) \) along with some algebra produces Equation 3 (see Reference 2 for details).

For example, if we want to demonstrate a reliability \( R \) of 90% at \( y = 1000 \) cycles with 90% confidence \( C \), then
A first variation on this is the case where there are six test times with each having progressed a variable number of cycles on test without failure. Consider the test values are \(250, 380, 428, 750, 618, 663\) cycles, and assume the Weibull distribution applies with \(\beta = 2\). Given 90% confidence, what is the reliability demonstrated at \(y = 275\) cycles? Using Equation 4, find the exponent to be 0.04263, which yields a reliability at \(y = 275\) cycles of 0.907, or about 91%. What \(B_p\) life is being demonstrated using this data? Using Equation 5 finds \(B_p \geq 730.8\) with 90% confidence.

Table 1 shows the sample size requirements for several values of \(m\) and the Weibull \(\beta\) using the “RC” (reliability – confidence) nomenclature. Practitioners should realize that, like the nonparametric zero-failure plans, the Weibull assumption zero-failure plans are very conservative. If we just meet the stated requirement including the assumptions, and we are using some confidence \(C\) (expressed as a decimal), then the probability of passing any test is \(1 - C\). Thus, with a zero-failure plan, the material or component being tested is hopefully better than the requirement we have set for it.

In a future Data Points installment, the test-planning discussion will continue with a look at Weibull plans that allow one or more failures.

**REFERENCES**


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### Table 1 — Sample Size Requirement; Zero Failure Weibull Plans; Reliability and Confidence Specified Using “RC” Nomenclature

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<th>R95C90</th>
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1. \(m\) is the ratio of test time, \(t\), to reliability requirement, \(y\). “RC” nomenclature indicates “Reliability” and “Confidence” used.

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