Building on Reliability: Reliability Test Planning, Part 2

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Q How does a Weibull statistical model help in reliability test planning?

A In our first article on this topic, we discussed how a test plan works: the test parameters, the type of data that results, and certain cases that are called nonparametric in that they do not depend on any assumed distribution for the failure mode.[1] In this part, we build further on these concepts and add to the mix the assumption of a Weibull statistical model for the underlying failure mode.

The Weibull distribution is a widely used model for failure phenomena in electromechanical components, systems, and materials testing. It is capable of modeling the three major classifications of failure modes that are commonly found: a) random, b) wear out or degradation, and c) early type failures called "infant mortality." Moreover, these three classifications are related to specific values of the Weibull distribution shape parameter, β. When failures are random, β = 1; when failures are wear-out or degradation related, β > 1; and when failures are *infant mortality*-related, β < 1.

Often, the starting point for reliability requirements demonstration is to assume *b* = 1. In the case of wear-out/fatigue types of failure modes, 1 < β ≤ 4 is commonly found. During an early product release/introduction or following some product modification, infant mortality conditions might occur, resulting in early failure for some units. In thoses cases, β < 1. This would typically occur in field applications. Generally, we do not test/validate for the infant mortality case, although in theory it is possible, as smaller values of β generally mean more variation in the life of a product.

When designing a test plan for the Weibull case, we have to assume a value of β. Generally, there may be good reason for any choice as there may be data (history) to draw from within a specific company, or there may be some public domain acknowledgment of a range of possible β values within a specific industry for a specific type of failure mode.

The standard two-parameter Weibull model has a closed-form cumulative distribution function described as:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad (1)$$

In Equation 1, *F(t)* is the cumulative failure probability at time *t*; β is the Weibull shape parameter (also called the Weibull slope) discussed earlier; and η is the Weibull characteristic life, or 63.2th percentile of the Weibull distribution model. Equation 1 is used, along with other theory, in developing test plans for the Weibull model.

RELIABILITY DEMONSTRATION PLANNING: WEIBULL MODEL, ZERO FAILURES

Recalling that if *n* units are tested for a time *t* and zero failures result, then using confidence *C*, the sample size requirement to demonstrate a reliability, *R*, would be:

$$n = \frac{\ln(1-C)}{\ln(R)} \qquad (2)$$

If the Weibull distribution is an assumption, Equation 2 is modified as shown in Equation 3:

$$n = \frac{\ln(1-C)}{m^{\beta}\ln(R)} \qquad (3)$$

In Equation 3, the variable *m* is the ratio of test time *t* to the time *y* at which reliability *R* applies. It is the "number of lives" the test time t is equivalent to, where a "life" is the life having reliability *R*. The assumed Weibull shape (slope) parameter is also needed. Equation 3 comes from the fact that there are zero failures allowed, and it uses the fundamental nonparametric relationship among reliability, confidence, and sample size, such that $R^n \geq 1 - C$. Then substituting the Weibull reliability function for *R(y)* along with some algebra produces Equation 3 (see Reference 2 for details).

For example, if we want to demonstrate a reliability *R* of 90% at *y* = 1000 cycles with 90% confidence *C*, then

Table 1 — Sample Size Requirement; Zero Failure Weibull Plans; Reliability and Confidence Specified Using "RC" Nomenclature[1]

	R90C90			R95C90			R99C90		
m	β=1	β=2	β=3	β=1	β=2	β=3	β=1	β=2	β=3
1.00	21.9	21.9	21.9	44.9	44.9	44.9	229.1	229.1	229.1
1.25	17.5	14.0	11.2	35.9	28.7	23.0	183.3	146.6	117.3
1.50	14.6	9.7	6.5	29.9	20.0	13.3	152.7	101.8	67.9
1.75	12.5	7.1	4.1	25.7	14.7	8.4	130.9	74.8	42.7
2.00	10.9	5.5	2.7	22.4	11.2	5.6	114.6	57.3	28.6
3.00	7.3	2.4	0.8	15.0	5.0	1.7	76.4	25.5	8.5

	R90C95			R95C95			R99C95		
m	β=1	β=2	β=3	β=1	β=2	β=3	β=1	β=2	β=3
1.00	28.4	28.4	28.4	58.4	58.4	58.4	298.1	298.1	298.1
1.25	22.7	18.2	14.6	46.7	37.4	29.9	238.5	190.8	152.6
1.50	19.0	12.6	8.4	38.9	26.0	17.3	198.7	132.5	88.3
1.75	16.2	9.3	5.3	33.4	19.1	10.9	170.3	97.3	55.6
2.00	14.2	7.1	3.6	29.2	14.6	7.3	149.0	74.5	37.3
3.00	9.5	3.2	1.1	19.5	6.5	2.2	99.4	33.1	11.0

1. *m* is the ratio of test time, *t*, to reliability requirement, *y*. "RC" nomenclature indicates "Reliability" and "Confidence" used.

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using Equation 2 without a Weibull assumption yields a sample size *n* of 22 units tested to *t* = 1000 cycles without failure. Suppose we assume a Weibull distribution for the failure mode, assume a modest wear-out mechanism with a Weibull β of 2, and are willing to test for twice as long (*t* = 2000 cycles), then *m* = *t*/*y* = 2000/1000 = 2, and using Equation 3 produces a new sample size *n* of 5.46 or 6. Essentially, this says that the Weibull assumption with β = 2 and twice the test time (2000) will demonstrate reliability at 1000 cycles with a four-fold decrease in sample size. Of course, we must test the smaller number of units for a longer period of time so there is the increasing risk of failure using the longer time.

A first variation on this is the case where a sample size has progressed without failure throughout a test, but the test times are variable. Then, with sample size *n*, the variable times t_i, and confidence *C* we can determine the reliability demonstrated at arbitrary time *y*, *R(y)*. The equation governing this case is Equation 4:

$$R(y) \ge (1-C)^{\frac{y^{\beta}}{\sum_{i=1}^{n} t_i^{\beta}}} \qquad (4)$$

Equation 4 can be rearranged to show the life demonstrated given a specified reliability *R*. This is referred to as the B_p life, where p is a percent and B_p is the life at which there is reliability (100 − *p*) %. For example, the B10 life is the life at which there is 90% reliability. This version with zero failures, confidence *C*, assumed Weibull β, and variable times is given in Equation 5 as:

$$B_p \ge \left(\frac{\ln(1-p/100)}{\ln(1-C)}\left(\sum_{i=1}^{n} t_i^{\beta}\right)\right)^{1/\beta} \qquad (5)$$

Now, suppose there are six test times with each having progressed a variable number of cycles on test without failure. Consider the test values are {250, 380, 428, 750, 618, 663}, and assume the Weibull distribution applies with β = 2. Given 90% confidence, what is the reliability demonstrated at *y* = 275 cycles? Using Equation 4, find the exponent to be 0.04263, which yields a reliability at *y* = 275 cycles of 0.907, or about 91%. What B_{50} life is being demonstrated using this data? Using Equation 5 finds $B_{50} \ge 730.8$ with 90% confidence.

Table 1 shows the sample size requirements for several values of m and the Weibull β using the "RC" (reliability – confidence) nomenclature. For example, R90C95 means a reliability requirement of 90% and confidence of 95%. To illustrate its use, suppose a reliability requirement of *R* = 95% at *y* = 1500 cycles using 90% confidence(R95C90). If a Weibull β of 3 is assumed and if we can test each item for *m* = 2 times the requirement (i.e., 2×1500=3000), then Table 1 shows the sample size n to be 5.6 or 6 units (rounded up).

Practitioners should realize that, like the nonparametric zero-failure plans, the Weibull assumption zero-failure plans are very conservative. If we just meet the stated requirement including the assumptions, and we are using some confidence *C* (expressed as a decimal), then the probability of passing any test is 1 − *C*. Thus, with a zero-failure plan, the material or component being tested is hopefully better that the requirement we have set for it.

In a future Data Points installment, the test-planning discussion will continue with a look at Weibull plans that allow one or more failures. ■

REFERENCES

1. Luko, Stephen, and Neubauer, Dean, Building on Reliability: Reliability Test Planning (Data Points), *ASTM Standardization News*, Vol. 49, No. 1, Jan./Feb. 2021, pp. 52-53.
2. Luko, Stephen, *Weibull Analysis with Assumed Weibull Slope Where NO Failures Are Observed,* SAE Technical paper 982032, International Off-Highway and Powerplant Congress and Exposition Milwaukee, Wisconsin, Sept. 14-16, 1998.