The Weibull Model — Building on Reliability

By Stephen N. Luko and Dean V. Neubauer

Q What is the Weibull distribution and how is it used in data analysis?

A In a previous Data Points column, "What Is Reliability," (Jan./Feb. 2018), we introduced its fundamental concepts.[1] In this installment, we build on the reliability concept by introducing the Weibull distribution and its use in data analysis.

The Weibull distribution is appropriate for many types of material properties as well as component life under a wide variety of conditions, including test scenarios and field issues. It can model failure rates for the three major classifications of failure modes: infant mortality or early failure, random failures, and wear-out failures.

Figure 1 depicts an idealized lifecycle failure rate referred to as the "bathtub" curve; the figure shows the three major failure mode regions and how each failure rate operates in that region. Early failure generally means that there is a failure mode, and some units have the failure condition more severely than others, leading to an increased propensity to fail early in those units, such as a manufacturing defect in a tire.

As failures are removed, and improvements are made, the failure rate decreases with time in this period. Eventually, the failure rate stabilizes and becomes constant for some useful life period. In this period, failures occur randomly and are often driven by external conditions such as a tire running over a nail and deflating or another foreign object causing damage during use.

As time marches on, products tend to lose robustness as aging and wear-out occur. This results in the increasing failure-rate portion of product life, such as performance with excessive treadwear where the tire fails.

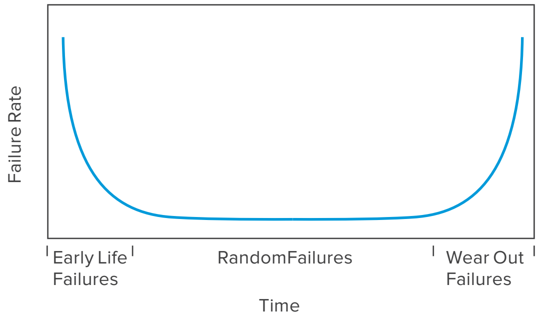


Figure 1 — The "Bathtub" Curve

The Weibull distribution is easy to work with and has a closed form cumulative distribution function, *F(t)*, as shown in Equation 1. It models variables on the domain (0,∞)—read as "time *t* can't be less than 0 but is open-ended on the right because we don't know when it will fail." Reliability is calculated as shown in Equation 2.

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}, t > 0 \quad (1)$$

$$R(t) = 1 - F(t) \quad (2)$$

In Equation 1, *F(t)* stands for the cumulative probability that a unit will fail by time *t*; the quantity *1-F(t)* is the survival probability, or reliability, at time *t* called R(t) in Equation 2. The Weibull has two parameters, η and β. The parameter β, also called the Weibull slope, is dimensionless and positive, and is typically referred to as the *shape parameter* because, for differing values of β and a fixed η, *F(t)* will have a different shape.

For example, when β = 1, the Weibull becomes the exponential distribution, and for β between 3 and 4, the Weibull approximately resembles a normal distribution. The term "Weibull slope" is also used by engineers because the Weibull *F(t)* can be linearized, and β is the slope of the resulting line. β also characterizes the scatter or dispersion in final test results. Higher values of β generally mean less scatter. The parameter η is called the *scale parameter* and has the same units as the variable t does. It turns out that *F* (η) = 0.632 for any β and, for that reason, the name "characteristic life" is also used for η.

The Weibull distribution can be linearized with a few simple algebraic operations. The following equation results:

$$ln\left\{-ln[1 - F(t)]\right\} = \beta ln((t) - \beta ln(\eta) \quad (3)$$

In Equation 3, the left-hand portion is seen to be a linear function of *ln(t)*. The slope of this line is β and the y-intercept is –β*ln*(η). In Figure 2, we show the theoretical linear function (3) for fixed η = 1000 and several values of β.

Observe how a higher β (slope) results in less variation or scatter. In a typical Weibull analysis, there is data in the form of failures and possibly "runouts" (suspensions or censored observations). The data is used with one of the well-known estimation procedures to estimate the parameters η and β. Once completed, the results (failures and estimated model) can be portrayed graphically using the Weibull probability plotting technique. This is typically done using any of several widely available software packages. In this example, we use

Minitab version 19. The resulting model is then used for various purposes such as reliability calculations or failure forecasting. The following example summarizes the method.

This example used $n = 25$ failures that had occurred in the field for a certain type of mechanical device used in the aerospace industry. This was a new/improved design being tried out in the field. Figure 3 shows the resulting probability plot. The analysis produced parameter estimates for beta (β), 3.084, and the characteristic life (η), 8472. The units here are operational cycles. Failure probability at time *t* can be read directly from the graphic, or we can use the Weibull distribution function (Equation 1) with the estimated parameters at various values of t. Table 1 shows an example of this. You can easily generate this table in Excel, where *F(t)* = 1-EXP(-1*(*t*/ β)^β) and values for η and β are referenced to their cell locations. Note that at *F(t)* = 63.2%, the characteristic life of η ~8472 is estimated. Also, with an estimated β of 3 we know that the failure mechanism is due to wear out.

t	*F (t)*	*R (t)*
50	1.34E-07	1.00E+00
100	1.13E-06	1.00E+00
500	1.62E-04	1.00E+00
1000	1.37E-03	9.99E-01
1500	4.79E-03	9.95E-01
2000	1.16E-02	9.88E-01
3000	3.99E-02	9.60E-01
4000	9.41E-02	9.06E-01

Table 1 – Field Data for Weibull; Failure Probability, *F(t)*; and Reliability, *R(t)=1-F(t)*, Estimates

This is not a safety-related issue, and the manufacturer has agreed to a warranty time of 1,500 hours. The table values show an estimated reliability at $t = 1500$ cycles of about 99.5%, assuring the manufacturer and the customer of this value. Note that point estimates of the Weibull parameters and associated reliability values are being used. These can be considered as approximate average values. If additional data sets are considered from the same product and running conditions, we would expect values like these, though they may vary (up or down) somewhat.

Readers interested in more about reliability are encouraged to consult the guide for general reliability (E3159). For analysis details, including the errors resulting from sampling, see References 2 or 3. For more about the use of reliability distributions for acceptance sampling, consult these two standards: the practice for factors and procedures for applying the MIL-STD-105 plans in life and reliability inspection (E2555) or practice for life and reliability testing based on the exponential distribution (E2696).

Figure 2 — The Weibull Cumulative Distribution Function *F(t)*, Linearized

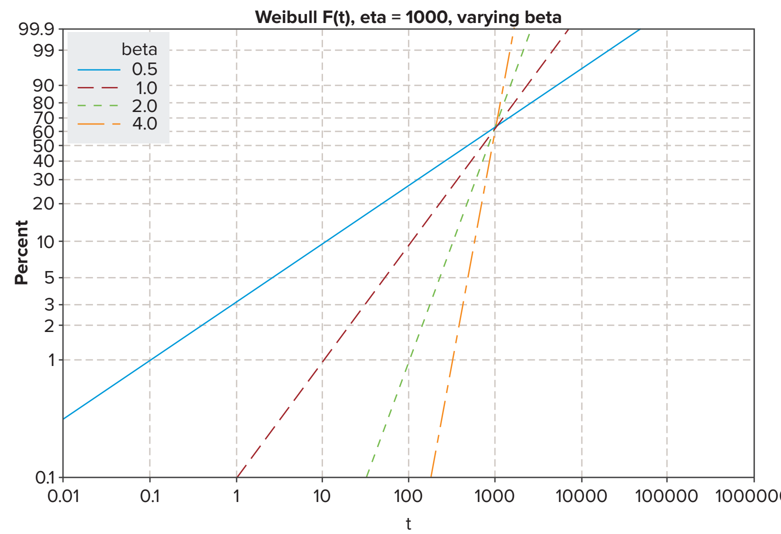
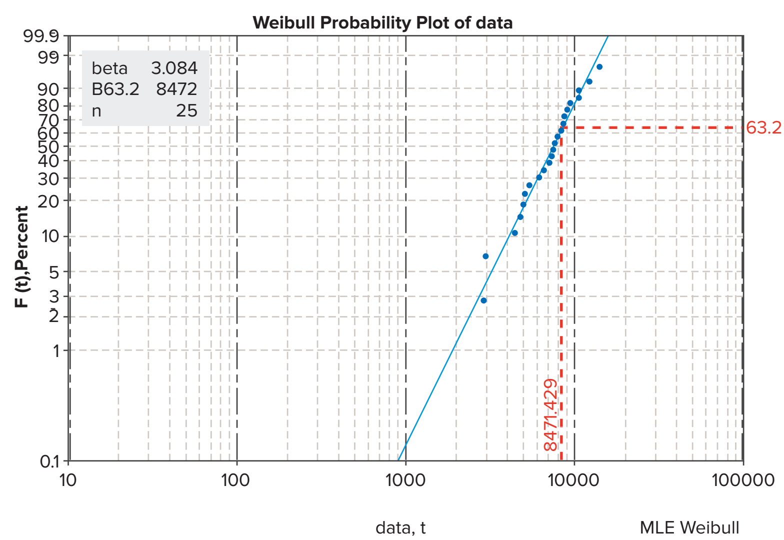


Figure 3 — Weibull Probability Plot of Aerospace Data



References

1. Luko, Stephen N., "What is Reliability–Key Concepts and Terminology," *Standardization News*, Vol. 46, No. 1, Jan./Feb. 2018, p. 28.
2. McCool, John I.,"Using the Weibull Distribution," *Reliability, Modeling, and Inference*. Wiley Series in Probability and Statistics, Hoboken, N.J., 2012.
3. Nelson, Wayne, *Applied Life Data Analysis*, John Wiley & Sons, New York N.Y., 1982.

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