Shewhart Individuals Control Charts

Part 1: Sample Size and Chart Reliability

BY THOMAS J. BZIK

Q: How does sample size impact Shewhart individuals control chart reliability?

A: ASTM E2587, Practice for Use of Control Charts in Statistical Process Control, documents methodologies available for standard control chart construction, including the familiar Shewhart individuals chart. E2587 recommends that a minimum sample size of 30 be used to construct a Shewhart individuals chart. This article examines the impact of sample size on reliability of the estimated control limits. What are the rewards of a larger sample size? What are the risks of using a smaller sample size than 30?

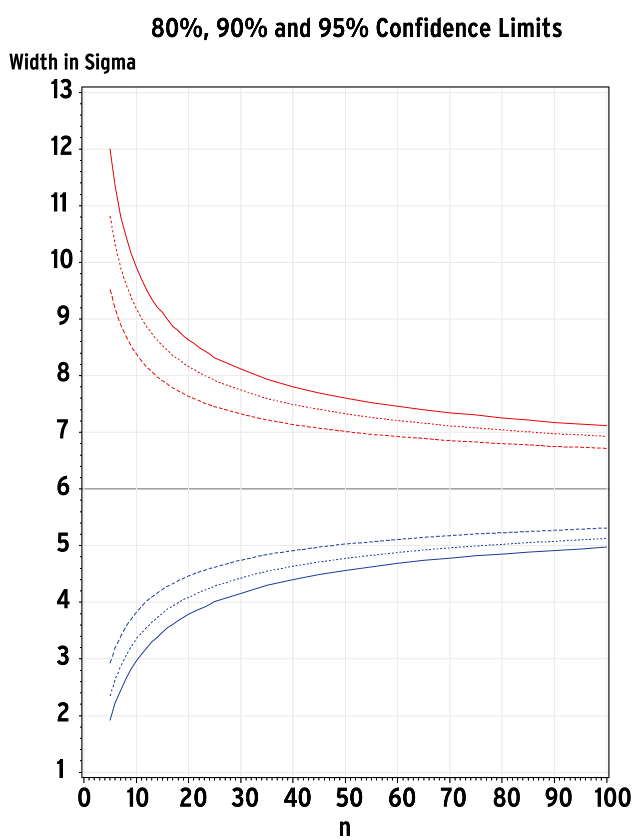

Figure 1

Figure 1 provides 80, 90 and 95 percent confidence intervals for the width of the distance between the upper and lower control limits expressed in terms of the true sigma as a function of sample size. This interval's expected width from a Shewhart individuals control chart, regardless of sample size, is given by ($\bar{x}$ + 2.659*$\bar{R}$) - ($\bar{x}$ - 2.659 $\bar{R}$), which is equivalent to ($\bar{x}$ + 3s) - ($\bar{x}$ - 3s) = 6s where s is the estimate of sigma from the data in question. In Figures 1 and 2, upper confidence limits are colored red and lower confidence intervals are colored blue. In addition, the outermost pair of confidence limits corresponds to 95 percent confidence, the middle pair of confidence limits corresponds to 90 percent confidence and the innermost pair of confidence intervals corresponds to 80 percent confidence. Observe in Figure 1 that for any confidence level, the interval's width gets closer to 6 sigma as sample size increases and conversely further from 6 sigma as sample size decreases.

For example, consider the 90 percent confidence level; observe that for n = 5, 20, 30 and 100 the intervals will be approximately (2.3, 10.8), (4.1, 8.2), (4.4, 7.7) and (5.1, 6.9), respectively. The choice of 90 percent confidence in interval construction implies that there is a 5 percent chance of obtaining a result below the confidence interval's lower limit and a 5 percent chance of obtaining a result above the upper limit. For example, for n = 5, there is a 5 percent chance that the width of a Shewhart individuals control chart is less than 2.3 sigma. Intuitively, this seems likely to be dangerous. What makes a sample size of 30 good enough but a sample size of 5 highly hazardous? Consider another example.

Figure 2 illustrates the impact that sample size has on the percentage of future points that will lie inside an estimated set of Shewhart individuals control chart limits (coverage probability). Normal theory says that asymptotically (as $n \to \infty$) when using 3 sigma limits, expect 99.73 percent of future points from an in-control process to lie inside the control charts limits and 0.27 percent to lie outside. Note that (1 – coverage probability) is the percentage of future points from an in-control process, which will be erroneously identified as being OOC (out of control). For any *n* less than infinity, the average coverage probability will be less than 99.73 percent and the alpha (Type 1 error rate) for the chart will be greater than 0.27 percent.

Again, for example, consider the 90 percent confidence level. Observe that for *n* = 5, 20, 30 and 100 the intervals will be approximately (0.71, 1.00), (0.95, 1.00), (0.97, 1.00) and (0.99, 1.00), respectively. For *n* = 5, 5 percent of the time the control chart will only include 71 percent or less of the data from an in-control process within its limit (29 percent or more OOC). This is yet another way of saying that a sample size of 5 is dangerously small. When *n* = 30 about 5 percent of the time, the coverage probability will be 97 percent or less (3 percent or more OOC). Note that the upper limits can barely reveal their distinctions within the limits of the graphical scale used in Figure 2. In an approximate manner the intervals that were much narrower than 6 sigma for Figure 1 are very likely to be the intervals in Figure 2 that yielded low coverage probabilities and high OOC in Figure 2. The intervals in Figure 1 that were much wider than 6 sigma were likely to be the intervals that gave rounded coverage probabilities that were very close to one for Figure 2.

Note in both figures that when *n* is small, additional data rapidly improves the reliability of the control limits, but this rate of improvement rapidly diminishes as *n* increases. The implications of these figures range from obvious to subtle. The ASTM E2587 recommendation of 30 is just that, a recommendation; it is not a point where things get suddenly better. The transition with sample size in the region of *n* = 30 is gradual. While using *n* = 5 or *n* = 10 in control chart construction is likely to involve significant hazard, what about using *n* = 20 or *n* = 25? Here things are not so clear-cut but depend on your tolerance for risk and its quality systems implications. For some, use of *n* = 20 may provide a reliable enough control-chart and for others, use of *n* = 30 may not be reliable enough for their purposes.

80%, 90% and 95% Confidence Limits

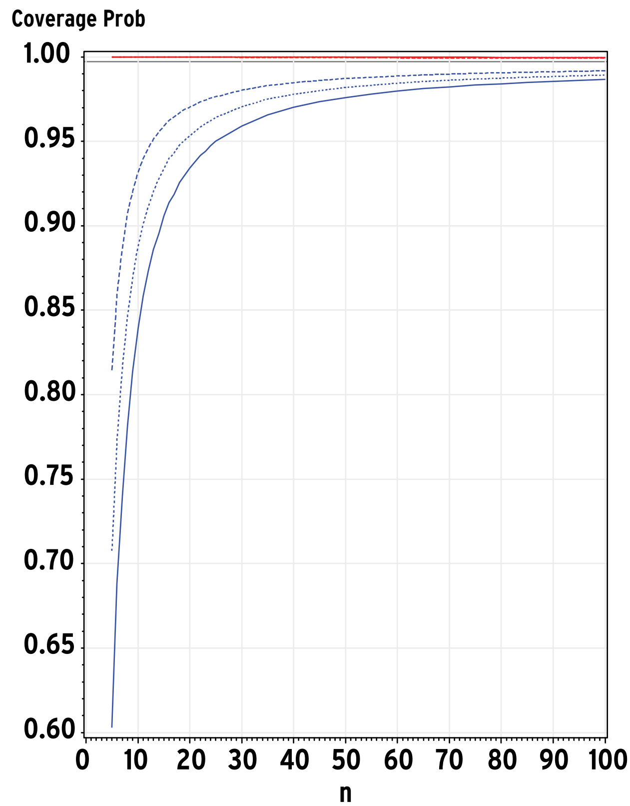

Figure 2

Readers are cautioned that the provided results are only applicable to Shewhart individual control charts that are constructed as per ASTM E2587.

Part 2 will further examine the implications of sample size on Shewhart individuals control chart reliability.

THOMAS J. BZIK, *Air Products and Chemicals Inc., Allentown, Pa., is the American Statistical Association representative to ASTM Committee E11 on Quality and Statistics; he serves as vice chairman of E11.20 on Test Method Evaluation and Quality Control and as member at large on the E11 executive subcommittee.*

DEAN V. NEUBAUER, *Corning Inc., Corning, N.Y., coordinates the DataPoints column; an ASTM fellow, he is chairman of Committee E11, chairman of Subcommittee E11.30 on Statistical Quality Control and chairman of E11.90.03 on Publications.*

Statistics play an important role in the ASTM International standards you write, and a panel of experts is ready to answer your questions about how to use statistical principles in ASTM standards. Please send your questions to SN *Editor in Chief Maryann Gorman at mgorman@astm.org or ASTM International, 100 Barr Harbor Drive, P.O. Box C700, West Conshohocken, PA 19428-2959.*