Meeting Material Specs

Part 2

By Joel Dobson

Q How do measurement error and process capability affect the probability that a product meets a specification? What is the relationship between parameter estimates in Statistical Process Control (SPC) and parameter estimates used in process capability? Where do the true values come from? Are these values derived from the sample?

A An earlier Data Points article, "Meeting Materials Specs" (May/June *Standardization News*)[1] considered the probability of a true value being out of specification, given that the measured value is in specification. Several questions were sent in by readers of that article regarding the relationship between process capability and SPC. This article attempts to clarify and builds on that article.

The probability of a true value being out of specification, given that the measured value is in specification is the consumer's risk. It was shown to depend on the values of the process capability indices, CPU and CPL, and the precision to tolerance ratio PTR. Equivalently, it can be expressed in terms of the process mean μ, overall observable process standard deviation σ, and the measurement standard deviation from gauge R&R.

This article considers also the producer's risk: the probability, for product that has true value in specification, that an observed value is out of specification, leading to rejection of the product.

We begin with definitions: LSL is the Lower Specification (Spec) Limit, and USL is the Upper Specification (Spec) Limit. Specification limits judge the raw data on a single item of product, because that data must be in spec for the material to ship.

The process capability indices related to LSL and USL are these three:

$$\text{CPL} = \frac{\mu - \text{LSL}}{3\sigma}$$

$$\text{CPU} = \frac{\text{USL-}\mu}{3\sigma}$$

$$\text{CPK} = \min\{\text{CPL,CPU}\} = \frac{\min\{\mu - \text{LSL,USL-}\mu\}}{3\sigma}.$$

where μ is the process mean and σ is the process standard deviation. We do not know the values of μ or σ, so we estimate them using the mean and standard deviation for the raw data.

Our estimate of σ is overall process standard deviation. It does not use the estimated process σ from a statistical process control (SPC) chart because these estimates are based on the average of rational subgroup standard deviation (or range) or average moving range (for I-charts, which plot individual measurements) and do not include all of the sources of variation needed for assessing process capability. Therefore, control charts, including I-charts, cannot be used in determining process capability.

The process capability CPK has to do with measured data being either in-spec or out-of-spec (OOS). The CPL and CPU might be thought of as one third of Z-scores for LSL and USL. Z-scores have the form

$$Z = \frac{X - \mu}{\sigma}$$

and represent the distance between a value X and the process mean μ in units of σ. More precisely, for USL and LSL we have:

$$Z_{USL} = \frac{\text{USL} - \mu}{\sigma} = 3 \cdot \text{CPU}$$
$$Z_{LSL} = \frac{\text{LSL} - \mu}{\sigma} = -3 \cdot \text{CPL}.$$

When we measure each item, we never know the true values (TV) because there is an error included within the measured values (MV). Using gauge R&R studies, we estimate the variance of the error term (ε), with τ^2 and $\hat{\tau}^2$ representing the true and estimated values of the measurement error variance, respectively. We also have an estimate of the variance of the measured values. So we are able to back out the variance of the true values. The equations that apply are these:

$$\text{MV} = \text{TV} + \varepsilon$$
$$\text{var}(\text{MV}) = \text{var}(\text{TV}) + \text{var}(\varepsilon) = \text{var}(\text{TV}) + \tau^2.$$

Solving for var(TV) and substituting the estimated values:

$$\widehat{\text{var}(\text{TV})} = \widehat{\text{var}(\text{MV})} - \hat{\tau}^2$$
$$\hat{\sigma}^2_{\text{TV}} = \hat{\sigma}^2_{\text{MV}} - \hat{\tau}^2.$$

If this estimated quantity is positive, we have an estimate that we can go forward with. We can also estimate the covariance of the true value and the measured value, which is done in the earlier article's two examples. Having the variances and covariance, we can make progress toward understanding their distribution. Assuming that the true values are independent of the measurement error, we have the covariance and correlation between MV and TV:

$$\text{cov}(\text{MV},\text{TV}) = \text{cov}(\text{TV} + \varepsilon, \text{TV}) = \text{var}(\text{TV}) + \text{cov}(\text{TV},\varepsilon) = \text{var}(\text{TV}) = \sigma^2_{\text{TV}} = \sigma^2_{\text{MV}} - \tau^2$$
$$\text{cor}(\text{MV},\text{TV}) = \frac{\text{cov}(\text{MV},\text{TV})}{\sqrt{\text{var}(\text{TV}) \cdot \text{var}(\text{MV})}} = \frac{\sigma^2_{\text{TV}}}{\sqrt{\sigma^2_{\text{MV}} \cdot \sigma^2_{\text{TV}}}} = \frac{\sigma^2_{\text{MV}} - \tau^2}{\sigma_{\text{MV}} \sqrt{\sigma^2_{\text{MV}} - \tau^2}} = \frac{\sqrt{\sigma^2_{\text{MV}} - \tau^2}}{\sigma_{\text{MV}}}$$

So, as the measurement error variance τ^2 decreases toward 0, the correlation between the true and measured values increases toward 1. Note that the covariance between two variables is their correlation coefficient times the product of their standard deviations.

Our idea is to consider that TV and MV follow a bivariate normal distribution. Granted, this may be an approximation. Figure 1 shows potential values of TV and MV in relation to the specification limits.[2] Values of either can be below LSL, between LSL and USL (thus in specification), or above USL. In the figure, green represents product for which both TV and MV meet the specification. Red represents product for which TV is out of specification, but measurement error causes the result to appear within specification. The consumer's risk is related to the fraction of product in these regions. Blue represents the opposite situation where TV is in-specification, but measurement error causes MV to be out of specification. This product therefore cannot be shipped and is a loss to the manufacturer. The white areas represent product that is correctly rejected. An ellipse is superposed to represent a bivariate distribution of TV and MV. The center of the bivariate normal distribution will be on the diagonal at a point (μ,μ) if the measurement process is unbiased. Usually, μ will not be at the center of the specification range.

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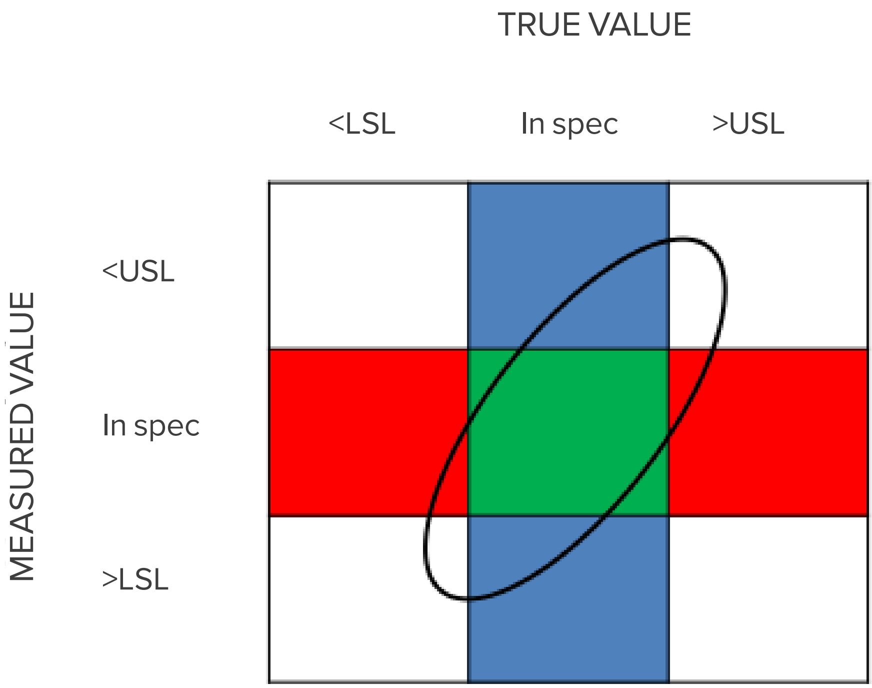

Figure 1 — Illustrating the bivariate normal distribution of true and measured values.

Conditional distributions from our bivariate normal relate to both the consumer's and producer's risk. The consumer's risk is the conditional probability that TV is outside the specification range, given that MV is inside. In the figure, this is P(red)/P(red+green). Similarly, the producer's risk is the probability, for product truly in-spec, that measurement is out of specification, so the product cannot be shipped P(blue)/P(blue+green).

We can use the probabilities calculated from the bivariate normal distribution to form our estimates for risks. As mentioned in the earlier article, the R function pmvnorm() from the R library mvtnorm can be used to estimate the relevant probabilities. Thus, probabilities related to meeting specification of interest to both the producer and consumer can be estimated. See the earlier article for example calculations. The estimate for TV variance is useful for estimating the producer and consumer risks but may not necessarily be useful for other applications.

REFERENCES

1. Dobson, Joel, "Meeting Material Specs," ASTM Standardization News, May/June 2017, Vol. 45, No. 3, p. 48, www.astm.org/standardization-news/?q=data-points/meeting-material-specs-mj17.html.

2. The author thanks Peter E. Fortini for his contribution of Figure 1 and related information used for this article.

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