Shewhart Individuals Control Charts

Part 2: More on Sample Size and Chart Reliability

BY THOMAS J. BZIK

Q: How does sample size impact Shewhart Individuals control chart reliability?

A: ASTM E2587, Practice for Use of Control Charts in Statistical Process Control, documents multiple methodologies available for standard control chart construction. A commonly used methodology is the Shewhart Individuals chart. E2587 recommends that a minimum sample size of 30 be used for the construction of a Shewhart Individuals chart. Part 1 of this DataPoints column examined the impact of sample size on control chart coverage probability and control chart width. Part 2 examines the impact of sample size on C_{pk} estimation. The description and use of C_{pk} and its related statistics are found in E2281, Practice for Process and Measurement Capability Indices. How well is C_{pk} estimated as a function of sample size? What are the rewards of having a larger sample size? What are the risks of using a smaller sample size than 30?

C_{pk} is a measure of process capability and serves as a figure of merit for the state of process control. C_{pk} is the number of times a three standard deviation wide interval will fit into the narrowest interval between the process mean and the nearest specification. For example, suppose $\bar{x}$ is 84.0, $\overline{MR}$ is 2.26, the lower specification is 70 and the upper specification is 92. The nearest specification to the mean is 92, and the width of the interval between the mean and closest specification is 92 - 84 = 8. The estimated three standard deviation interval using a subgroup size of one in individuals chart construction has a width of 2.66*$\overline{MR}$ = 6.0. C_{pk} is calculated as 8/6 = 1.33. From a process control perspective, the larger the C_{pk} the better.

Figures 1, 2 and 3 each provide 80 percent, 90 percent and 95 percent confidence intervals for the estimated C_{pk} as a function of sample size using a true C_{pk} of 0.75, 1.00 and 2.00, respectively, in each figure's construction. The departure from the horizontal reference line in each figure representing the true C_{pk} used in the construction of each figure shows the uncertainty resulting from using sample sizes between 5 and 100. In all figures, upper confidence limits are colored red and lower confidence intervals are colored blue. In addition, the outermost pair of confidence limits corresponds to 95 percent confidence, the middle pair of confidence limits corresponds to 90 percent confidence, and the innermost pair of confidence intervals corresponds to 80 percent confidence.

In all figures, for any selected confidence level, the confidence interval width narrows around the true C_{pk} as the sample size increases. Also, the intervals show marked asymmetry about the true C_{pk} for small *n* with the upper confidence limits being further away than the lower confidence limits. Note that Figures 2 and 3 are fairly similar to one another in relative shape and spacing. Approximate additional figures can be constructed for any true $C_{pk} > 2$ by multiplying the axis of Figure 3 by the desired true $C_{pk}/2$.

Observe in Figure 1 that all the upper confidence limits for C_{pk} are above 1 when the true $C_{pk} = 0.75$ and *n* is 12 or less. This strongly argues against using too small a sample size when estimating C_{pk}. Note in all figures that the change with respect to sample size has started to gradually level out by *n* = 30. Consider that the ASTM recommendation of 30 is a recommendation and not a point where things get suddenly better. With a higher tolerance for risk, perhaps a small *n*, such as 20, may provide an acceptable tradeoff. With a lower tolerance for risk, a sample size of 30 may be insufficient.

Practitioners using the figures in Parts 1 and 2 can determine the tradeoffs between sample size, control limit uncertainty, control chart coverage probabilities and C_{pk} uncertainty. Knowing the tradeoffs allows a rational decision to be made about acceptable sample size. If the

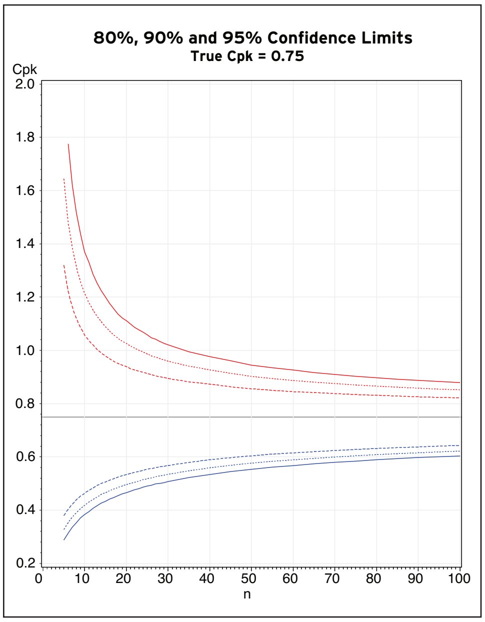

Figure 1

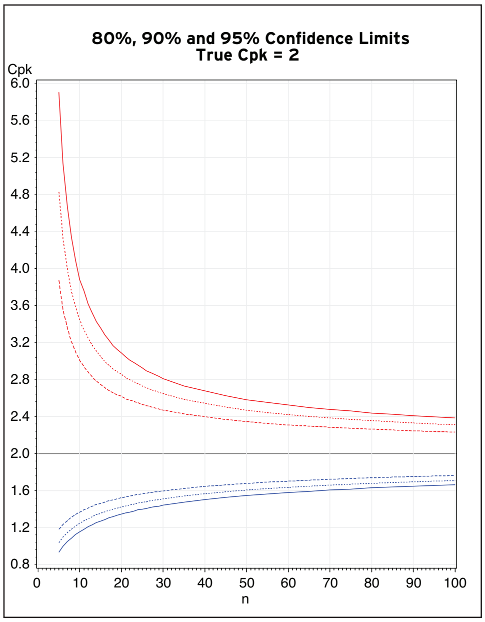

Figure 3

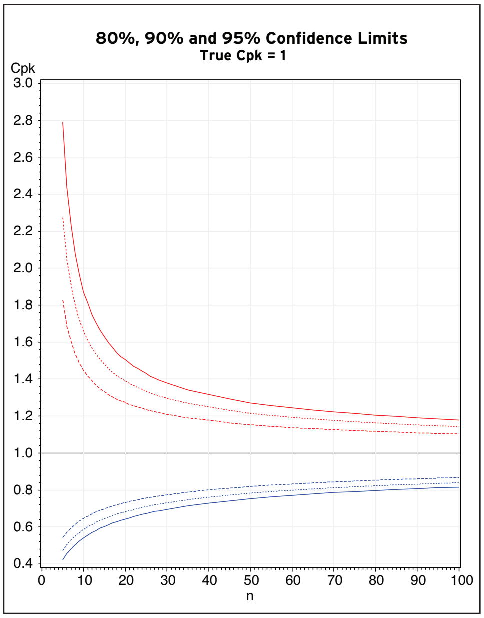

Figure 2

risk of using a smaller sample size is acceptable, the subsequent use of a larger "training" data set (e.g., 30 or more) should be considered after additional "training" data has become available. Such updating will mitigate some of the hazard of employing sample sizes smaller than 30.

Readers are cautioned that the provided results only apply to Shewhart Individuals control charts constructed as per E2587.

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