Easy Estimation of Measurement Uncertainty

E2554 and Control Chart Techniques

By Neil Ullman

Q How can we easily obtain an estimate of measurement uncertainty?

A Many committees have recognized a need to incorporate information in standards about establishing uncertainty estimates. This has generally been developed with references to the "Guide to the Expression of Uncertainty in Measurement,"[1] which involves attempting to identify all possible contributions to variation of tests.

ASTM Committee E11 on Quality and Statistics has developed an alternate approach using data that is often obtained in the normal use of the test method. The principle assumed here is that uncertainty is an intermediate precision that falls between the short-term repeatability and the broader reproducibility measured with interlaboratory studies.

Form and Style for ASTM Standards, Section A22, refers to how ASTM regards measurement uncertainty. Since those statements were written, E11 approved E2554, Practice for Estimating and Monitoring the Uncertainty of Test Results of a Test Method Using Control Chart Techniques. For many test methods this is a practical way both to develop such estimates and to observe changes to the precision of the method within the laboratory.

The practice assumes that some reference or stable material is available to test over time. The results are initially examined using traditional types of control charts that seek to see if the process is in control as described in ASTM E2587 or Manual 7.[2] But often there is additional variation over time that can be incorporated while estimating uncertainty.

This discussion focuses on Example 1 from E2554, where samples of three dosimetry units are tested at nine different times. (Since more samples are desirable, a revision to E2554 has been proposed to do so.)

The data is listed in Table 1 along with two statistics computed for each sample:

— The range, which is a measure of the variation within the sample or repeatability. It is simply the difference between the largest and the smallest readings; and

— The average or mean of the readings.

RANGE CONTROL CHART

The first consideration for examination is the repeatability. The standard suggests directly working with standard deviations, but ranges are also mentioned in a note and are considered

Table 1—Example Data from E2554

Test No.	Rep 1	Rep 2	Rep 3	Range	Mean
1	0.282	0.274	0.276	0.008	0.2773
2	0.294	0.274	0.284	0.020	0.2840
3	0.3	0.284	0.292	0.016	0.2920
4	0.29	0.3	0.292	0.010	0.2940
5	0.296	0.294	0.297	0.003	0.2957
6	0.29	0.278	0.284	0.012	0.2840
7	0.29	0.29	0.29	0.000	0.2900
8	0.278	0.288	0.286	0.010	0.2840
9	0.284	0.292	0.292	0.008	0.2893

an excellent substitute when small samples are taken at each time period. In Figure 1, the ranges are graphed in time order. Two values are computed and also plotted:

— The average range, $\overline{R}$ = (Sum of the ranges)/(number of samples) = 0.087/9 = 0.0097 (plotted with the solid line)

— The upper and lower control limits computed as

$UCL = D_4 * \overline{R} = 2.575 * 0.0097 = 0.0249$ (plotted with a dotted line)

$LCL = D_3 * \overline{R} = 0 * 0.0097 = 0$

The points look randomly dispersed around the center line with no values near the upper limit. We assume the variation within samples is basically constant for all the samples, and we can then estimate repeatability standard deviation from the average range using the following formula:

Estimate of repeatability standard deviation = $\bar{R}/d_2$ = 0.0097/1.693 = 0.0057

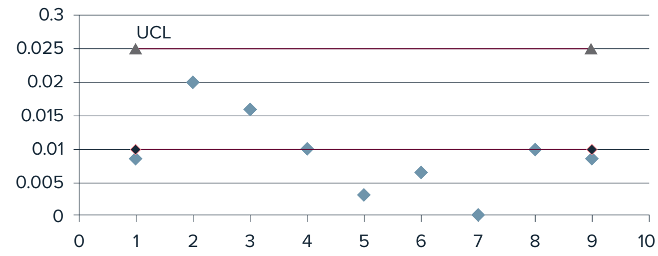

Figure 1 — Range Control Chart

MEANS CONTROL CHART

The means for each sample are also graphed in Figure 2. The overall mean, $\overline{\overline{X}}$ is found as 0.2878 and plotted as a solid line. Then control limits are computed and shown with dotted lines. The control chart limits are found as:

$UCL_{mean} = \overline{\overline{X}} + A_2 * \overline{R} = 0.2878 + 1.023*0.0097 = 0.2977$

$LCL_{mean} = \overline{\overline{X}} - A_2 * \overline{R} = 0.2878 - 1.023*0.0097 = 0.2779$

This chart has only a few points, but there are two of the nine readings that are questionable. The first sample is below the lower limit and the fifth is almost at the upper limit. These are 3 sigma limits based on repeatability and we would expect values at the limits only about once in 400 samples. So there appear to be other causes for variation over time.

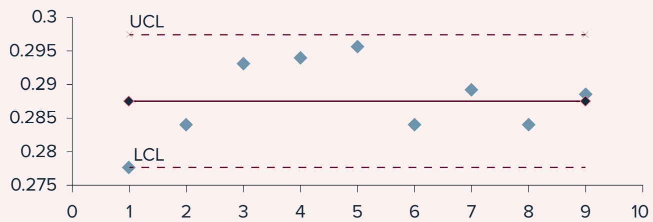

Figure 2 — Means Control Chart

We can compute the variation among the nine sample means and find this as $S_{\bar{x}} = 0.0059$. This result actually consists of two pieces:

S_{time}
The variation between times, and

$\frac{S_{repeat}}{\sqrt{n}}$

The variation of the sample averages based on the repeatability of measurement, where n is the number of tests in the sample, in this case, three.

These are combined through the squares of these standard deviations. And we can compute the value of s_{time} as follows:

$$S_{time} = \sqrt{S_{\bar{x}}^2 - \frac{S_{repeat}^2}{n}} = \sqrt{0.0059^2 - \frac{0.0057}{3}} = 0.0049$$

"This chart has only a few points, but there are two of the nine readings that are questionable. The first sample is below the lower limit and the fifth is almost at the upper limit."

UNCERTAINTY ESTIMATE

We can now estimate the uncertainty for individual results, which consists of one time and one test. This is represented as s_u and is found as:

$$S_u = \sqrt{S_{time}^2 + S_r^2} = \sqrt{0.0049^2 + 0.0057^2} = 0.0075$$

Thus, the variation among individuals taken over time would be greater than those taken under repeatability conditions. However, we also need to recognize that this estimate has been developed with a limited number of samples and over longer time periods would probably be greater.

UNCERTAINTY CONTROL CHART

Having arrived at this new estimate of variation we can modify the control chart. Control chart limits are based on three standard deviation intervals around the overall average. Since we have samples of three tests we estimate what the standard deviation would be for a random time and the sample average. Thus we compute a new estimate of standard deviation associated with the variation over time.

$$S_{u-aves} = \sqrt{s_{time}^2 + \frac{s_r^2}{n}} = 0.0059$$

Using this estimate, the new control chart limits are now:

$$UCL_{mean} = \overline{\overline{X}} + 3 * S_{u\text{-}aves} = 0.2878 + 0.0117 = 0.3055$$

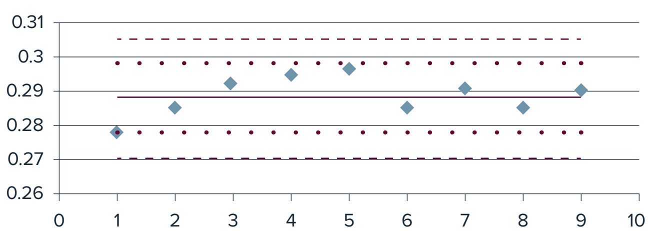

Figure 3 — Uncertainty Control Chart

$$LCL_{mean} = \overline{\overline{X}} - 3 * S_{u\text{-}aves} = 0.2878 + 0.0117 = 0.2701$$

The new outer limits are the uncertainty control limits. Figure 3 shows that all the points are now well within the uncertainty limits, but this is because we have forced the limits to accommodate the between sample variation. The importance is not for how this chart looks now, but rather for how this performs in the future. If the tests continue to be conducted and stay within the larger uncertainty control limits we can argue that the estimate of uncertainty standard deviation remains as reported.

There are then two other possibilities:

— If there are out of control indications, it is important to investigate and possibly increase the reported uncertainty value.

— If the points begin to fall tighter together it is a very positive sign that perhaps we have improved the process and the uncertainty has become smaller.

“If the tests continue to be conducted and stay within the larger uncertainty control limits we can argue that the estimate of uncertainty standard deviation remains as reported.”

REFERENCES

1. International Organization for Standardization (ISO), ISO Guide 98, “Guide to the Expression of Uncertainty in Measurement,” Geneva, Switzerland, 1995.

2. E2587, Practice for Use of Control Charts in Statistical Process Control; Manual 7, Presentation of Data and Control Chart Analysis.

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