DATA POINTS

A STATISTICS Q&A

Inspection Redundancy – Part 1

Why multiple or redundant inspections make sense.

By Stephen N. Luko

Q What is the purpose of multiple or redundant inspections?

A In general, inspection processes for a defined attribute, a single inspection of an item with the attribute, may result in classifying that item as conforming, or not having the attribute, for any number of reasons. In this case it is common to say that the attribute has "escaped" the inspection process. On repeating the inspection several times, it may happen that the attribute would be discovered or detected and dispositioned as nonconforming, or as having the attribute. In this situation we say that there is a principle of inspection redundancy at play. There is a greater likelihood of finding the attribute when it is really there when several redundant inspections are made.

Note that finding the attribute in any inspection is generally not just a function of the "size" of the attribute (as it is in traditional POD — probability of detection — applications for material flaws). Rather, it is more a function of other nuisance variables such as skill of the inspector, position of the inspection device, condition of the environment during the inspection and other nuances of the inspection process. When redundant inspections are made we improve the probability of uncovering undesirable attributes when they are present because we are using multiple trials. We can still speak of detection probability in multiple trials of an inspection process but rename this "inspection composite probability of detection" (ICPOD) so that it is not confused with the more traditional probability of detection.[1]

In a redundant inspection process there is a probability p that any single inspection will discover the attribute when it is really present in the item. If several, say n, inspections are independently performed, the inspection process works like a binomial distribution with success probability p. Here, success means that the inspection has discovered the attribute. Let X be the random variable that counts the number of successful discoveries of an attribute in repeating the inspection n times under identical conditions on the same unit. In working with attribute inspection, what is desired is the event "$X \geq 1$". That is, when X is at least 1 then the inspection process will have uncovered the attribute. In the language of probability we want $P(X \geq 1)$ to be reasonably high, and this can be made so by redundancy of inspection.

The principle of redundancy means that multiple inspections have a greater chance of finding an attribute than any single inspection does. In an imperfect inspection process, redundancy of inspection will improve the detection probability of the attribute. The principle of "at least one" and its sister concept "zero defects" are very useful in general applications of the redundancy principle to inspection processes. The relationship between the concepts of at least one and zero defects may be cast in terms of probability as follows.

$$P(X \geq 1) = 1 - P(X=0) \qquad (1)$$

In Equation 1, X is a random variable of the attribute type equal to the number of findings of some attribute in an inspection process. The left-hand side means the probability that X is at least 1; the right-hand part is one minus the probability that $X = 0$, or no attributes are found. The random variable X has a binomial distribution with parameters p and n, where p is the probability that a single inspection will discover an attribute and n is the number of independent inspections. The principles of at least one and zero defects work together in this application.

REDUNDANCY AND ICPOD

Suppose there is an inspection process for some attribute that has a discovery probability of p. Then, p is the probability of finding an undesirable attribute or condition (mistake, error, flaw, etc.) on one inspection when the object actually has the undesirable condition. Generally, p may be estimated by experiment or by using an industry standard for the specific application. In cases where this is difficult or uncertain, a conservative value of p may be chosen by experts in the applied area. Values such as 0.5 are not uncommon. In some cases, the value of p may actually be changing from inspection to inspection. Under this condition the value of p should be considered as an average value over many inspections.

When p is selected, this means that there is a probability of 1 - p that any single inspection will fail to find the condition when it is present. So, what happens when two such inspections are done – independently? In two inspections, one desires at least one of them to detect the undesirable condition. The ICPOD for two inspections is calculated using Equation 1, where $X = 0$ means that the inspection fails to find the condition on two inspections. In this example, ICPOD for the two inspections means at least one of the two inspections finds the attribute. For two independent inspections, the probability of failing to find the condition on both inspections is $(1 - p)^2$, where the binomial distribution for two independent trials has been used. Then the ICPOD is:

$$ICPOD = P(X \geq 1) = 1-(1-p)^2 \quad (2)$$

If $p = 0.8$ for a single inspection, then for two inspections ICPOD increases to $1-(1-.8)^2 = 0.96$. More generally, for n inspections the inspection composite POD is calculated as:

$$ICPOD = P(X \geq 1) = 1-(1-p)^n \quad (3)$$

For example, with $n = 3$ independent inspections, ICPOD increases, in theory, to 0.996. For $p = 0.5$, $n = 4$ inspections are required to give an ICPOD of better than 90 percent (using Equation 3 with $p = 0.5$ and $n = 4$). Care should be exercised when using this theory in some applications. This is true for cases where inspections may be correlated from one time to the next. That would violate the independence of inspections being used above.

We can use Equation 3 to solve for n given p and any desired final ICPOD. The solution is Equation 4:

$$n = \frac{ln(1\text{-}ICPOD)}{ln(1\text{-}p)} \quad (4)$$

For example, if ICPOD = 0.99 is desired and p is about 0.6, then a redundancy of about $n = 5$ will satisfy the ICPOD requirement.

If several inspections are used, each inspection process may have a different inspection probability of detection for a given attribute. This can happen due to inspections at differing locations, differing inspection devices, different operators, training differences or other variables that are outside of control. Part 2 of this series will take up this variation of inspection.

RERERENCE

1. Brown, Jennifer, "Accessing Crack Detection Capability - Statistical Models in Practice," ASTM *Standardization News*, Jan./Feb. 2015, pp. 14-16; www.astm.org/standardization-news/images/ma15/ma15_datapoints.pdf.

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